1. ***Mr. Lincoln's Line***

Mr. Lincoln's 32 fifth grade students are standing in line to go to lunch.
The 3rd child, and every 3rd child after, is a boy.!
The 5th child, and every 5th child after, is wearing glasses.!
The 4th child, and every 4th child after, is carrying a lunch box.
1. What do you know for certain about the 20th child in line? 2. What do you know about the 24th child?
3. What do you know about the 30th child?

The Problem

Explain how you found your answers.

Answer Check

Extra: How many children would need to be in this line for us to be certain there was a boy wearing glasses and carrying a lunch box? Explain how you know.

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually **get** the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

The 20th child is wearing glasses and carrying a lunch box. If your answer **doesn't** match ours,

- did you try skip-counting on a number line or number grid?
- did you make a list of the numbers you get when you count by 3, 5, and 4?
- did you realize that more than one thing might be true about the 20th, 24th and 30th children?
- did you check your arithmetic?

  If anything on that page helps you, you might *revise* your answer, and then leave a *comment* that tells us what you did. If you're still stuck, leave a *comment* that tells us where you think you need help.

  If your answer **does** match ours,
- did you remember to find the answers to 2 and 3?
- is your explanation clear and complete?
- did you try the Extra question?
- did you verify your answer with another method?

Revise your work if you have any ideas to add. Otherwise leave us a *comment* that tells us how you think you did—you might answer one or more of the questions above.

Method 1: Make a List

After reading the problem I decided to make a list from 1 to 32 so I could think about the children who were standing in line. After making my list I put a check next to the 3 and I counted three after that and I kept putting checks. I did that for 4 but I put an 0 and I did it for 5 but I put an X:

1
2 3√ 40 5X 6√ 7 80 9√

10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32
X √0
√X 0
√
0X √
√0 X
√ 0
√X 0

Now that I had a list I just checked each of the numbers:

1. 20th child has a lunchbox and wears glasses 2. 24th child is a boy and has a lunchbox
3. 30th child is a boy and wears glasses

Method 2: Solve a Simpler Problem

That seems like a lot of kids to think about and so I just thought about 10 kids in line. I drew a picture to think about it:

I wondered if there would be any boys who wore glasses or carry lunchboxes because the first 10 kids didn't have anything in common. I thought about continuing to count by 3s and count by 4s and count by 5s and when any of those numbers might match. It didn't seem so hard to count and so next I made these lists:

count by 3s: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30 count by 4s: 4, 8, 12, 16, 20, 24, 28, 32 count by 5s: 5, 10, 15, 20, 25, 30

I noticed that 20 showed up in my 4s and my 5s list and so that kid had a lunchbox and glasses.

I noticed that 24 showed up in my 3s and my 4s list and so that kid was a boy and had a lunchbox. I noticed that 30 showed up in my 3s and my 5s list and so that kid was a boy wearing glasses.

Method 3: Make a Table of Multiples

I made a table listing all the multiples of 3 (every 3rd child), 4 (every 4th child), and 5 (every 5th child) up to 32.

3s	**4s**	**5s**
Boy	*Lunch Box*	*Glasses*
3	4	5
6	8	10
9	12	15
12	16	20
15	20	25

1. I looked for columns that included a 20. It was in the list of 4s and 5s, so the 20th child had a lunch box and glasses.
2. The number 24 was in the list of 3s and 4s, so I knew he was a boy and carried a lunch box.
3. Number 30 was listed in the 3s and 5s column, so I knew it was a boy wearing glasses.

Extra: I looked at my list and couldn't find any number that appeared in all three lists. I looked in the lists of 3s and 4s for any number with 0 or 5 in the ones place, since they would be multiples of 5. 15 and 30 were in the 3s list, but not in the 4s list. 20 was in the 4s list, but not in the 3s. So I knew the class had to be larger than 32 students.

I made my lists of multiples longer by continuing the 4s first, because there would be fewer of them, until I found even multiples of 5, those ending in 0. Then I added to the list of 3s to find a match. I had to continue this process until I found 60, which was the first number to be on all three lists.

3s	**4s**	**5s**
Boy	*Lunch Box*	*Glasses*
3	4	5
6	8	10
9	12	15
12	16	20
57		
60		

Method 4: Draw Pictures

I made this number line:

And I put a • above each number that is a boy. I put a √ above each number that was a kid carrying a lunch box. I put a Δ above each number that is a kid wearing glasses. It looked like this:

Now I looked at what I put over the 20 and I saw that the kid in line there wears glasses and has a lunch box. The kid in line at spot 24 is a boy and carries a lunch box. The kid in line at spot 30 is a boy wearing glasses.

Method 5: Skip Count

I made a list of the numbers from 1 to 32. I put a B, for boy, above the 3. Then I skip counted by 3s and put a B above every 3rd number. For the children wearing glasses I counted by 5s and put a G above each number I counted. I counted by 4s and put LB above each number I counted to show the children with lunch boxes.

9. I looked at the number 20. It had G and LB above it, because I landed on it when I counted by 4s and 5s, so that student wore glasses and had a lunch box.
10. Above number 24 I had written B and LB, so I know it was a boy carrying a lunch box. I landed on it when I counted by 3s and 4s. I noticed that the 12th child also was a boy with a lunch box.
11. Number 30 had B and G, because I landed on it counting by 3s and 5s. That was a boy with glasses.

Extra: If a child was going to be a boy with glasses and a lunch box, I had to find a number with B, G and LB on it. I didn't have any on the numbers up to 32, so I had to continue counting by 3s, 4s, and 5s, putting letters on the numbers I counted. I tried it up to 50, but I still didn't have any number with all three codes. I kept skip counting up to 70, first with 3s and then with 4s. When I counted by 5s, I stopped at 60 because that had all three letters. So the 60th child was a boy wearing glasses and carrying a lunch box.

Method 6: Direct Use of Multiplication and Number Theory

I used what I know about factors and multiples to solve the problem. I know that the multiples of 5 all end in 5 or 0. This means that the 20th and 30th children both wear glasses. 4 is a factor of 20 and 24, but not 30. 3 is a factor of 24 and 30, but not 20.

12. I know the 20th child wears glasses and carries a lunch box because 4 and 5 (but not 3) are both factors of 20.
13. The 24th child is a boy with a lunch box, because 3 and 4 are factors of 24, but 5 is not.
14. The 30th child is a boy wearing glasses, because 3 and 5 are factors of 30, but 4 is not.

Extra: I knew I needed to find the number that is the lowest common multiple of 3, 4, and 5. All multiples of 5 end in 5 or 0. All multiples of 4 are even. I checked the even multiples of 5, adding the digits to find ones divisible by 3. 30 was the lowest, but it's not a multiple of 4. 60 is divisible by 3, 4, and 5, so the line would have to have 60 children to be sure that there was a boy with glasses and a lunch box.

I could also think about it like this:

I knew I needed to find the number that is the lowest common multiple of 3, 4, and 5. Since they have no common factors among them, their LCM is $3 \bullet 4 \bullet 5 = 60$. The line would have to have 60 children to be sure that there was a boy with glasses and a lunch box.

Method 7: Direct Use of Multiplication and Number Theory Organized in a Table

I'll find out if each child is a multiple of 3, 5, or 4. If it is a multiple of 3, 5, or 4, the child will be a boy, wear glasses or carry a lunch box or a mix.

1. # child | Every | How | Yes/No ----------
-
20 | 3rd | 20/3 = 6.6....| N 20 | 5th | 20/5 = 4 | Y 20 | 4th | 20/4 = 5 | Y
20th = glasses + lunch box
2. # child | Every | How | Yes/No ----------
-
24 | 3rd | 24/3 = 8 | Y 24 | 5th | 24/5 = 4.8 | N 24 | 4th | 24/4 = 6 | Y
24th = boy + lunch box
3. # child | Every | How | Yes/No ----------
-
30 | 3rd | 30/3 = 10 | Y 30 | 5th | 30/5 = 6 | Y 30 | 4th | 30/4 = 7.5 | N
30th = boy + glasses

Extra:

60 is a multiple of 4 because 4 x 15 equals 60. 60 is a multiple of 5 because 5 x 12 equals 60. 60 is a multiple of 3 because 3 x 20 equals 60. Since 60 is a multiple of 4, 5 and 3, the 60th child will be a boy with glasses and carrying a lunch box.

This problem requires a strong conceptual understanding of factors and multiples. Children who have not mastered these ideas formally can still solve the problem using methods that lay the groundwork for more sophisticated thinking. Younger children might apply these concepts by using a number line or grid or by drawing pictures, and then skip-counting to identify the attributes of the children listed.

When we first offered this problem, many children made lists of the multiples of 3, 4, and 5, and then looked for the lists that numbers 20, 24, and 30 appeared in. Others used their direct knowledge of factors and multiples to analyze 20, 24, and 30.

Teaching Suggestions

Resist the urge to give direct instructions on a specific approach. Ask students to paraphrase the problem to check on their understanding before they begin working on it. Depending on where your students are in their thinking of factors and multiples, you might choose to act out this problem or a simpler version. Another idea would be to use a combination of a drawing or number line and manipulatives to mark the child in line that has a lunchbox or wears glasses or is a boy. Ask questions that help your students understand the language of the problem, visualize it, and discover patterns. Good questions help students clarify their thinking and give you useful information as well. The questions in the Answer Check, above, might serve as good prompts to help students make progress. Encourage students to use a strategy

that works for them. You can see from the various methods that we have thought to use for this problem that there are many ways to approach this problem. And, we may not have thought of them all.

Sample Student Solutions

focus on

Completeness

Novice	**Apprentice**	**Practitioner**	**Expert**
Has written very little that explains how the answer was achieved.	Submitted explanation without work or work without explanation. Leaves out enough details that another student couldn't follow or learn from the explanation.	Explains all of the important steps taken to solve the problem, which might include explaining: • how they used their picture or table or list. • any relationships used. • the rationale behind each decision they made.	Adds in useful extensions and further explanation of some of the ideas involved. The additions are helpful, not just "I'll say more to get more credit."