



Math Fundamentals PoW Packet

Missing Pages

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Welcome!

This packet contains a copy of the problem, the “answer check,” our solutions, and teaching suggestions.

We invite you to visit the PoW discussion groups to explore these topics with colleagues. From the Teacher Office use the link to “PoW Members” or use this URL to go to *funpow-teachers* directly: <http://mathforum.org/kb/forum.jspa?forumID=526> [Log in using your PoW username/password.]

The Problem

In *Missing Pages* solvers use what they know about consecutive numbers to find the page numbers on a missing leaf in a book. In the Extra they learn about page-numbering conventions and apply that knowledge to think about possible page numbers on a missing leaf.

The text of the problem is included below. A print-friendly version is available using the “Print this Problem” link on the problem page.

Missing Pages

Mr. Dawkins asked his class to open their math books to a new lesson. Drew reported that a page was missing from his book. When Mr. Dawkins asked him what page it was, Drew told him that the sum of the page numbers on either side of the missing page was 219.

What are the page numbers on the missing page?

Explain how you solved the problem and show how you know your answer is correct.

Extra: Alexa reported that a page was missing from her book, too, and that the numbers on the pages on either side of the missing one added to 129. Drew thought for a few seconds, looked in several books in his desk, and said that a sum of 129 was not very likely.

What did Drew discover about page numbering? How did he know 129 was not possible?

Explain your thinking.



Answer Check

After students submit their solutions, we encourage them to check their answers by looking at the answer that we provide. Below is what they will see. You might use the accompanying questions as prompts to help students who are struggling, or to encourage those who have found a correct solution to improve their explanation.

One of the missing pages is 110. Now you can find the other one.

If your answer **doesn't** agree with ours —

- did you remember that one page is missing, with a number on each side?
- did you remember that the numbers on the visible facing pages add to 219?
- did you try to find the two numbers that are on the missing page?
- did you check your arithmetic?

Did you realize that the page numbers on right-hand pages of any book are always odd numbers, and those on left-hand pages are always even numbers?

If you used guess-and-test, did you tell . . .

- what numbers you tried?
- how you knew whether they worked or not?
- how you decided what to try next?

If your answer does match ours,

- is your explanation clear and complete?
- did you show how you know your answer is right?
- did you try the Extra?
- did you have any “Aha!” moments or notice any patterns? Describe them.

Our Solutions

Here are several ways I imagine children might solve the problem. They are not meant to be prescriptive or comprehensive. We often receive solutions from students who have used approaches we've not anticipated. I hope you will share such approaches on the *funpow-teachers* discussion board, along with any teaching strategies you found to be successful.

Strategy 1: Guess-and-test

I thought that the two missing pages should add up to close to the visible pages. Half of 219 is about 110, so I started checking with that number as the visible left-hand page, making the missing page numbers 111 and 112, and the visible right page number 113. That produced a sum that was too high, so I tried 109 as the visible left page number. I noticed that decreasing the visible left page number by one gave a difference of 2 in the visible total, so I knew 108 should be the correct visible left number.

Visible left	Missing right	Missing left	Visible right	Sum of visible pages
110	111	112	113	$110 + 113 = 223$ (too high by 4)
109	110	111	112	$109 + 112 = 221$ too high by 2)
108	109	110	111	$108 + 111 = 219$ Yes!

The page numbers of the missing page are 109 and 110.

[Students might look in a real book and add numbers they find.]

Strategy 2: Find visible left page number from the difference

I knew that in 4 consecutive pages the last page would have a number 3 more than the first page. I subtracted 3 from the sum to find double the number of the first page. Half of that is the number on the left-hand visible page.

$$219 - 3 = 216$$

$$216/2 = 108$$

The missing page contains pages 109 and 110. The right-hand visible page is 111.

Check: The four consecutive pages are 108, 109, 110, 111.

$$\text{The two visible pages} \quad 108 + 111 = 219$$

Extra: I used my same strategy to find out what the missing pages would be.

$$129 - 3 = 126$$

$$126/2 = 63 \quad \text{This would be the starting number.}$$

The four pages would be 63, 64, 65, and 66. The sum of 63 and 66 is 129.

I looked in some books and found that page 63 is always on the right side. The left-hand page numbers are all even numbers. So 129 could not be the sum of the visible pages.

Strategy 3: Logical reasoning, halving the sum

There are 4 consecutive page numbers. The second number, which is the right-hand side of the missing page, is one **more** than the first number, the left-hand visible page. The third number is one **less** than the fourth, which is the right-hand visible page. So the sum of the two inside (missing) pages is the same as the sum of the two visible facing pages, or 219.

Half of 219 is 109.5, so the two missing pages are 109 and 110

Check: [as above]

Extra: I used the same strategy to find what the missing pages would be. Their sum would be the same as the sum Alexa gave for the facing pages.

Half of 129 is 64.5, so the two missing pages would be 64 and 65.

I looked in some books and found that 64 and 65 are never on the same page. The odd number pages are always on the right and the even numbers are always on the left.

Strategy 4: Algebra

I use n to represent the number on the left-hand visible page, the first of the 4 consecutive pages. The other pages would be $n + 1$, $n + 2$, and $n + 3$. The sum of the first and fourth pages is 219.

$$n + (n + 3) = 219$$

$$2n = 216 \quad \text{combine like terms}$$

$$n = 108 \quad \text{the left-hand visible page number}$$

The missing pages are 109 and 110.

[Check as above.]

Teaching Suggestions

Some solvers, especially younger ones, may try looking in books to help conceptualize the problem. The problem uses the word *page* to mean both the leaf that is missing from the book, and the individual side of a leaf. Since this is the common usage, we hope it doesn't cause confusion for children who might think very literally. If it does, some explanation from the teacher might be in order.

The problem is an opportunity to explore patterns in consecutive numbers and how they are related to each other. Number lines could be useful for thinking about these.

The **Extra** extends the thinking of the main problem. The conventional way that pages are numbered in published books — even numbers on left-hand pages (called *verso*) and odd numbers on the right (called *recto*) — might be a new idea for students. Can you find any published books that are an exception to that?

The Extra provides an opportunity for more advanced learners to explore modular arithmetic, sometimes called *clock arithmetic*. It may appear obvious that the sum of any two consecutive pages must be odd, since the sum of an odd number and an even number is always odd. We now know, however, that not all odd numbers can be the sum of the numbers on the front and back of one leaf. What then must be true about any odd number that does work? Think division and remainders. There are resources on the Online Resources Page that can help you further.

The Online Resources Page for this problem also contains links to related problems in the Problem Library and to other web-based resources. If you would like one page to find all of the 2009-2010 Current Problems as we add them throughout the season, consider bookmarking this page:

<http://mathforum.org/pow/support/>

Scoring Rubric

The problem-specific scoring rubric we use to assess student solutions is a separate stand-alone document available from a link on the problem page. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work. A **generic student-friendly rubric** can be downloaded from the *Teaching with PoWs* link in the left menu (when you are logged in). We encourage you to share it with your students to help them understand our criteria for good problem solving and communication.

We hope these packets are useful in helping you make the most of Math Fundamentals Problems of the Week. Please let me know if you have ideas for making them more useful.

~ Claire

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