Mentor Instructions

Football Fanfare

http://mathforum.org/pows/

In ***Football Fanfare*** the key concepts are area of a rectangle and factors. Most Math Fundamentals students will have the most difficulty keeping their numbers organized. We are hoping that they use the Make a Table strategy to help them be organized and systematic but it may not be their first inclination. Although we've included a method that makes use of prime factorization, we would be surprised if this method were used by students.

Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually **get** the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

> There are 12 possible rectangular seating arrangements. Two of them are 6 rows of 18 seats and 18 rows of 6 seats.

If your answer **doesn't** match ours,

- have you checked your arithmetic?
- have you re-read the problem?
- did you eliminate some pairs because you thought they weren't reasonable in a real stadium? If so, please tell us about that.

> If any of those ideas help you, you might *revise* your answer, and then leave a comment that tells us what you did. If you're still stuck, leave a *comment* that tells us where you think you need help.

If your answer **does** match ours,

- did you answer the second question?
- did you try the Extra?
- are you confident that you could solve another problem like this successfully?
- did you include each step you took to solve the problem?
- is your explanation clear and complete?
- did you make any mistakes along the way? If so, how did you find them?
- are there any hints that you would give another student?

> *Revise* your work if you have any ideas to add. Otherwise leave us a *comment* that tells us how you think you did—you might answer one or more of the questions above.

Scoring Rubric

The problem-specific scoring rubric we use to assess student solutions is a separate stand-alone document available from a link on the problem page or from Lesson 5 of the Online Mentoring Guide. Be sure to download it and refer to it when scoring.

Some important ideas about separating the categories:

- A student can choose a good strategy without interpreting the problem correctly. If the student interprets the problem incorrectly, yet picks a good strategy based on that interpretation, she/he is a Practitioner in Strategy (despite the fact that the strategy will never result in a correct solution).
- Similarly, a student can be accurate without getting the problem right. Evaluate Accuracy based on whether the chosen strategy was executed correctly – even if the strategy is faulty.

It's important to separate these categories; they are not entirely dependent on one other.

Mentoring Suggestions

To score Practitioner in Strategy using a guess-and-test method, the solver needs to test numbers systematically – achieving success through skill and understanding, not pure luck. Random guessing, or guesses that are not reasonable in the context of the problem, indicate an Apprentice.

Students need to show most of their numbers and key calculations as well as their rationale in order to be considered Practitioner in Completeness. Students may use text, numbers, or ideally some combination of both to represent their processes. Mentors may want to suggest or model effective representation and notation that can help improve the Clarity of an explanation:

- Lists, tables and number sentences (equations) often help the reader follow more easily than does pure text. A student might benefit from an example of how to type equations.
- Starting in about fifth grade it's preferable to use the asterisk (*) as the multiplication symbol, so as to avoid confusion with x used as a variable. Asterisks appear on many calculators and are used in spreadsheets and in many popular published curricula.
- The slash (/) is an effective way to indicate division.
- Be on the lookout for students who string equations together, e.g., 19,708 - 19,087 = 621/9 = 69. This creates an incorrect statement. Mentors should model more conventional ways of representing multiple calculations.

Solutions may vary in the degree of formality with which they solve the problem. When judging the Clarity of students' writing, consider their age/grade. When scoring use your best judgment to determine whether the math language is "level-appropriate," but don't hesitate to provide suggestions or models that the students can use to improve their communication, including appropriate math terms: subtract, minus (rather than "take away"), and difference, as well as correct place value terminology.

Be sure to look down through the thread to see if the student has changed an answer after looking at ours. If they have, in order to score Practitioner in Strategy and Completeness, the solver's strategy must be compatible with the corrected answer and their explanation must include enough detail for you to be confident they understand the answer and have not simply changed it to match ours. A specific reference to how or why they changed an answer can be counted as reflection.

Be on the lookout for (and encourage!) exceptional insights and observations regarding number patterns and relationships, which might lead to Expert status in Interpretation and/or Completeness. Solving or checking the answer by another means can boost a score in Strategy and/or Reflection.

Our Solutions

Method 1: Make a Table

I know after reading the problem that there are 108 square cards and I need to arrange them in a rectangle with an area of 108 square units. I know that the area of a rectangle is length x width. I made a table to think about the different lengths and widths a rectangle with that area might have.

length	width	possible dimensions
1	108	1 by 108 **or** 108 by 1
2	54	2 by 54 **or** 54 by 2
3	36	3 by 36 **or** 36 by 3
4	27	4 by 27 **or** 27 by 4
5	21.6	doesn't work
6	18	6 by 18 **or** 18 by 6
7	15.43...	doesn't work
8	13.5	doesn't work
9	12	9 by 12 **or** 12 by 9
10	10.8	doesn't work
11	9.81...	doesn't work

I could continue but I notice that I've already found the width that goes with a length of 12 and so I've found all of the pairs. There are twelve possible arrangements.

I've never been to a game that had card stunts but I have seen some on television. From the ones that I've seen, they are usually a little longer than they are tall but they're not too skinny. So, I would pick the 18 (length) by 6 (width) or the 12 (length) by 9 (width) and if I have to make a decision, I'll go with the 12 by 9 as my final answer!

Method 2: Number Theory and Make a Table

I notice that

- the rectangular area is 108 square units.
- there are 108 volunteers to hold cards.
- each card must be 1 square unit in area.
- 108 is an even number
- using what I know of divisibility rules, 108 is divisible
  - by 2 because it's even
  - by 3 because 1+0+8=9 and that's divisible by 3
  - by 4 because the last two digits (08) form a number divisible by 4
  - not by 5 because it doesn't end in 0 or 5
  - by 6 because it's divisible by both 2 and 3
  - by 9 because the sum of the digits is divisible by 9
  - not by 10 because it doesn't end in 0
  - by 12 because it's divisible by both 4 and 3

I don't remember the rules for 7 or 8 or 11 and so I just tried dividing 108 by those three numbers and none work evenly. I have enough information to make a table to think about the length and width.

length	**width**
1	108
2	54
3	36
4	27
6	18
9	12
12	9

I notice that I have both 9 by 12 and 12 by 9 in my table. I realize that I could also list the others that way and so now I have:

length	**width**
1	108
2	54
3	36
4	27
6	18
9	12

length	**width**
12	9
18	6
27	4
36	3
54	2
108	1

There are 12 different arrangements.

For the second question I don't think I want anything too skinny in length or too skinny in width and so I'd use one of the arrangements in the middle of the list in my table. I also would want to think about if I want to make a drawing (like the happy face example) or if I want to spell something (like the "T" example). To make a final decision I would model my idea using graph paper.

Extra: 500 is almost 5 times as much as 108, so I'm going to guess that there will be 60 possible arrangements for a 500-card stunt (there were 12 for a 108-card stunt, and 5 • 12 = 60). But there might be fewer...

Now I'll figure out exactly how many there are by looking at the factor pairs:

rows • columns	rows • columns
1 • 500	500 • 1
2 • 250	250 • 2
4 • 125	125 • 4
5 • 100	100 • 5
10 • 50	50 • 10
20 • 25	25 • 20

What a surprise! There are exactly the same number of arrangements! Maybe this is because of the prime factorization:

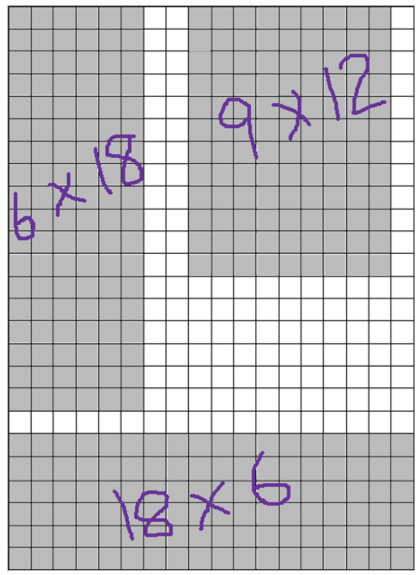
$108 = 2^2 \bullet 3^3$

$500 = 2^2 \bullet 5^3$

Interesting, they have the same number of prime components - so there are the same number of ways of combining them.

Method 3: Draw a Picture

We've been using graph paper to think about different sizes of rectangles and so I decided to draw a picture to think about what's happening in this problem. Here's my picture:



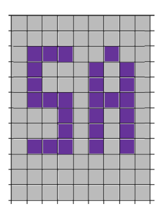
As I was drawing these three possibilities, I saw that there were pairs of rectangles since I could draw a 6 by 18 but I could also draw 18 by 6. I didn't have enough room on my paper but I know that I could draw a 12 by 9 to go with the 9 by 12.

I didn't want to draw a 1 by 108 or 108 by 1 but I know that's another possibility because the area of that rectangle is also 108 square units. I also know that these would work:

2 by 54 and 54 by 2
3 by 36 and 36 by 3
4 by 27 and 27 by 4

I found 12 different arrangements.

For Question 2 I thought it would be fun to think about how my initials might fit and I picked the 9 x 12 to try it in. Here's how it looks!



Method 4: Listing Factor Pairs

I know from the problem that 108 square units is the area of the rectangle and I have to find all the possible lengths and widths that might work for the pep squad. If I think of the lengths and widths as the factors of 108, I think I can find all the possibilities.

We've been learning about factors and how to use the divisibility rules to know if a number is a factor of another number. I'm going to list them:

length	1	2	3	4	6	9	12					
width	108	54	36	27	18	12	9					

I skipped

- 5 because 108 doesn't end in 0 or 5.
- 7 and 8 because I used my calculator and they don't divide evenly into 108
- 10 because 108 doesn't end in 0
- 11 because I know my 11's and if 10•11 is 110, 11 can't be a factor of 108

I see that 9 is both a length and a width and so I finish my factor list like this:

length	1	2	3	4	6	9	12	18	27	36	54	108
width	108	54	36	27	18	12	9	6	4	3	2	1

I found 12 different arrangements.

To answer Question 2 I think you have to think about what you want to show up on the cards and how much space you have in the stadium. If it's the Super Bowl stadium you don't have to worry about the space and so maybe I'd pick 1 (tall) by 108 (wide) but that would be a design and not a message.

Method 5: Use Prime Factors

To figure out this problem, I knew I needed to find all of the factors of 108. This would tell me how many rows and columns they could use to make the card stunts.

I decided that I would find the prime factors of 108 and then use them to find all the combinations of two factors to find all the combinations of rows and columns. Here is how I thought about it. The last one lists all the prime factors of 108.

3 • 36 = 108
3 • 3 • 12 = 108
3 • 3 • 2 • 6 = 108
3 • 3 • 2 • 2 • 3 = 108

Now all I needed to do was find all of the combinations using the prime factors. I made a list of the possible seating arrangements in rows and columns (seats in each row).

Rows	Columns	Prime Factorization
1	108	[1] [2 • 2 • 3 • 3 • 3]
108	1	[2 • 2 • 3 • 3 • 3] [1]
2	54	[2] [2 • 3 • 3 • 3]
54	2	[2 • 3 • 3 • 3] [2]
3	36	[3] [2 • 2 • 3 • 3]
36	3	[2 • 2 • 3 • 3] [3]
4	27	[2 • 2] [3 • 3 • 3]
27	4	[3 • 3 • 3] [2 • 2]
6	18	[2 • 3] [2 • 3 • 3]
18	6	[2 • 3 • 3] [2 • 3]
9	12	[3 • 3] [2 • 2 • 3]
12	9	[2 • 2 • 3] [3 • 3]

There are 12 different arrangements possible.

Extra: I decided to figure it the same way as I figured the first part of the problem. The first set of factors is 1 and 500 (easy). Then I took 2 and 250 and found all the prime factors of them.

2 • 250 = 500
2 • 2 • 125 = 500
2 • 2 • 5 • 25 = 500
2 • 2 • 5 • 5 • 5 = 500

Next I used all the prime factors and again found all the combinations to find the possible seating arrangements.

Rows	Columns	Prime Factorization
1	500	[1] [2 • 2 • 5 • 5 • 5]
500	1	[2 • 2 • 5 • 5 • 5] [1]
2	250	[2] [2 • 5 • 5 • 5]
250	2	[2 • 5 • 5 • 5] [2]
4	125	[2 • 2] [5 • 5 • 5]
125	4	[5 • 5 • 5] [2 • 2]
5	100	[5] [2 • 2 • 5 • 5]
100	5	[2 • 2 • 5 • 5] [5]
20	25	[2 • 2 • 5] [5 • 5]
25	20	[5 • 5] [2 • 2 • 5]
10	50	[2 • 5] [2 • 5 • 5]
50	10	[2 • 5 • 5] [2 • 5]

Interestingly again there are 12 possible arrangements.

Please let me know if you have any questions about this, or if you'd like to discuss any part of it.

~ Suzanne suzanne@mathforum.org